CORRECTION CENTRALE

Centrale 86

Question 1.

* Calcul de K, f.

K : $U_N = r I_N + E = r I_N + K \Omega_N \Rightarrow \boxed{K = \dfrac{U_N - r I_N}{\Omega_N}}$

AN $K = \dfrac{12 - 20 \cdot 0{,}1}{100\pi} = \boxed{3{,}18 \cdot 10^{-2}\ V.s/rad}$

f : $f \Omega_N^2 = \Gamma_{em} \Omega_N = K I_N \Omega_N \Rightarrow \boxed{f = \dfrac{K I_N}{\Omega_N}}$

AN $f = \dfrac{3{,}18\,10^{-2} \times 20}{100\pi} \simeq \boxed{2 \cdot 10^{-3}\ mNs/rd}$

Question 2

* Eq. dif. ensemble moteur + charge

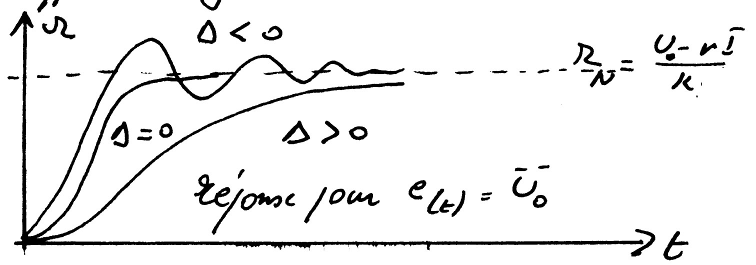
$$f\Omega + J\frac{d\Omega}{dt} = \Gamma_{em} \quad ; \quad U = r i + \ell \frac{di}{dt} + E \quad (1)$$

$$f\Omega + J\frac{d\Omega}{dt} = K i \quad ; \quad U = r i + \ell \frac{di}{dt} + K\Omega$$

$$i = \frac{f}{K}\Omega + \frac{J}{K}\frac{d\Omega}{dt} \rightarrow \frac{di}{dt} = \frac{f}{K}\frac{d\Omega}{dt} + \frac{J}{K}\frac{d^2\Omega}{dt^2}$$

$$\boxed{U = \frac{J\ell}{K}\frac{d^2\Omega}{dt^2} + \left(\frac{rJ + \ell f}{K}\right)\frac{d\Omega}{dt} + \left(\frac{rf}{K} + K\right)\Omega}$$

* Types de régime transitoire.



Δ > 0 régime apériodique
Δ = 0 " critique
Δ < 0 " oscillatoire amorti

* Calcul de $\Omega(t)$ et $i(t)$ pour $r = 0$ et $f = 0$
(moteur à résistance d'induit = 0 et couple de frottements = 0 ($C_r = 0$))

$$U = \frac{J\ell}{K}\frac{d^2\Omega}{dt^2} + K\Omega \quad ; \quad \frac{KU}{J\ell} = \frac{d^2\Omega}{dt^2} + \frac{K^2}{J\ell}\Omega$$

Sans second membre. $\dfrac{d^2\Omega}{dt^2} + \dfrac{K^2}{J\ell}\Omega = 0$

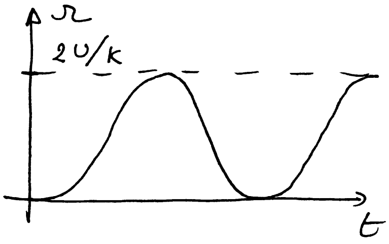
$$r^2 + \frac{K^2}{J\ell} = 0 \Rightarrow r = \pm j\sqrt{\frac{K^2}{J\ell}} \quad \text{(racines complexes conj.)}$$

$$\Omega(t) = A\cos\frac{Kt}{\sqrt{lJ}} + B\sin\frac{Kt}{\sqrt{lJ}} + \frac{U}{K}.$$

à $t = 0$ $\quad \Omega = 0 \quad A = -\frac{U}{K}$

$\quad \frac{d\Omega}{dt} = 0 \quad B = 0$

$$\Omega = \frac{U}{K}\left(\underbrace{1 - \cos\frac{K}{\sqrt{lJ}}\cdot t}_{+2 \atop 0}\right)$$



$$i = \frac{J}{K}\frac{d\Omega}{dt}$$

$$i(t) = \frac{J}{K}\cdot\frac{U}{\cancel{K}}\cdot\frac{\cancel{K}}{\sqrt{Jl}}\sin\frac{K}{\sqrt{Jl}}\cdot t = \boxed{\frac{U}{K}\sqrt{\frac{J}{l}}\sin\frac{K}{\sqrt{Jl}}\cdot t}$$

* Condensateur modélisant l'ensemble moteur

$$i = \frac{J}{K}\frac{d\Omega}{dt} = \frac{J}{K}\frac{d\,E/K}{dt} = \frac{J}{K^2}\frac{dE}{dt} \equiv C\frac{du}{dt} = i_C$$

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$\boxed{C = \frac{J}{K^2}}$ $\quad$ AN : $E = K\Omega \Rightarrow K = \frac{120}{50\pi} = 0{,}764$ Vs/rad

$$C = \frac{1}{0{,}764^2} = \boxed{1{,}71\ F}$$

Question 3

* Équation permettant de calculer $U(t)$ et $\Omega(t)$

$i(t) = I$ $\quad$ (1) devient $\begin{cases} u(t) = rI + K\Omega(t) & (2) \\ f\Omega(t) + J\frac{d\Omega}{dt} = KI \rightarrow \Omega + \frac{J}{f}\frac{d\Omega}{dt} = \frac{K}{f}I \end{cases}$

$$\Omega(t) = Ke^{-t/\tau} + \frac{K}{f}I \quad \text{avec } \tau = \frac{J}{f}$$

A $t = 0$ $\quad \Omega_{(0)} = 0 \quad K + \frac{K}{f}I = 0$

$$\boxed{\begin{aligned} &\Omega(t) = \frac{K}{f}I\left(1 - e^{-t/\tau}\right) \quad \text{avec } \tau = \frac{J}{f} \\ &u(t) = rI + \frac{K^2 I}{f}\left(1 - e^{-t/\tau}\right) \end{aligned}}$$

AN $\quad K = \frac{E_N}{\Omega_N} = \frac{120}{50\pi} = 0{,}764$; $f = \frac{K I_N}{\Omega_N} = \frac{0{,}76 \times 50}{50\pi} = 0{,}243$

$$\tau = \frac{J}{f} = 4{,}1\ ms.$$

$$\Omega_{(t)} = 157{,}2\left(1 - e^{-t/\tau}\right) \quad \text{avec } \tau = 4{,}1.10^{-3}\,s.$$

$$u_{(t)} = 10 + 120\left(1 - e^{-t/\tau}\right)$$

Graphes de $u(t)$ et $\Omega(t)$

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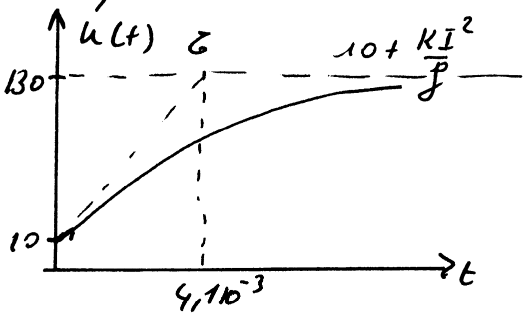
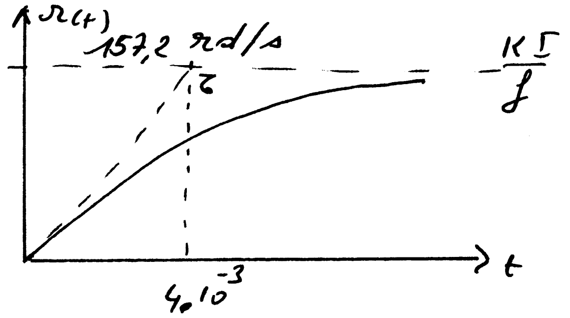
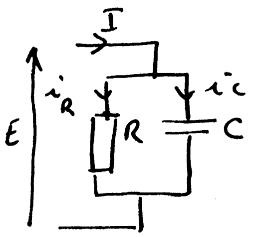




* Démonstration que le moteur est équivalent à un condensateur C en // avec une résistance R.



$$\left.\begin{array}{l} E = K\Omega = R\,i_R \\ i_C = C\dfrac{dE}{dt} = CK\dfrac{d\Omega}{dt} \end{array}\right\} \quad I = i_R + i_C = \frac{K}{R}\Omega + CK\frac{d\Omega}{dt}.$$

$$\left.\begin{array}{l} \dfrac{RI}{K} = \Omega + CR\dfrac{d\Omega}{dt} \\ \dfrac{K}{f}I = \Omega + \dfrac{J}{F}\dfrac{d\Omega}{dt} \end{array}\right\} \quad \begin{array}{l} RC = \dfrac{J}{F} \\ \dfrac{R}{K} = \dfrac{K}{f} \end{array}$$

A comparer avec (seconde ligne)

$$\boxed{R = \frac{K^2}{f} \qquad C = \frac{J}{K^2}}$$

AN : $C = 1{,}71\ F$

$R = 2{,}4\ \Omega$